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## Angular Momentum and the Aircraft-Store Separation Problem

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### Nomenclature

$F_{xA}, F_{yA}, F_{zA}$	= forces along the $X_A, Y_A, Z_A$ axes
$F_{xB}, F_{yB}, F_{zB}$	= forces along the $X_B, Y_B, Z_B$ axes
$I_x, I_y$	= rolling and pitching moments of inertia
$I_{x2}, I_{y2}$	= rolling moment of inertia of outer body and inner body
$I_{y1}, I_{y2}$	= pitching moment of inertia of outer body and inner body
$K_1, K_2$	= complex constants defined by the initial conditions
$m$	= total configuration mass
$m_1, m_2$	= masses of outer and inner bodies
$M_{p\alpha}$	= Magnus moment stability derivative
$M_q + M_{\alpha}$	= pitch damping moment stability derivative
$M_{xA}, M_{yA}, M_{zA}$	= moments about the $X_A, Y_A, Z_A$ axes
$M_{xB}, M_{yB}, M_{zB}$	= moments about the $X_B, Y_B, Z_B$ axes
$M_{\alpha}$	= pitching moment stability derivative
$p, q, r$	= angular velocity components resolved along the $X, Y, Z$ axes
$s$	= gyroscopic stability factor
$t$	= time
$\dot{u}, \dot{v}, \dot{w}$	= body linear velocity components resolved along $X, Y, Z$ axes
$X, Y, Z$	= right handed coordinate system when $X$ is the missile axis of symmetry
$x_1$	= distance from c.g. of outer body to total configuration c.g.
$x_2$	= distance from c.g. of inner body to total configuration c.g.
$Z_{\alpha}$	= normal force stability derivative
$\alpha$	= Complex angle of attack
$\lambda_{1,2}$	= dynamic damping factors
$\omega_1, \omega_2$	= nutation and precession frequencies

Received May 10, 1973; revision received May 30, 1973. Acknowledgement is due to C. J. Cohen, Research Associate for the Warfare Analysis Department. Equation (2) is due to him.

Index category: LV/M Dynamics, Uncontrolled.

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### Subscripts

$A$	= aeroballistic axis system
$B$	= body fixed axis system
$1$	= outer body
$2$	= inner body

### Introduction

AIRCRAFT-store separation is a serious consideration for both weapon and aircraft designers. The weapon designer wants to avoid the large release disturbance since it affects the weapons accuracy. The aircraft designer wants to avoid the large release disturbance since it is dangerous to the pilot and can result in damage to the aircraft.

There have been various methods of improving weapon separation characteristics but none has been entirely satisfactory since they are based largely upon trial and error. Present systems are rather erratic in their separation characteristics due to variation in store configurations.

In retrospect, it is quite unlikely that a store separation problem existed when aircraft flew at 250 knots and dropped 2000 lb bombs due to the inertia loads being much greater than the aerodynamic loads. The bombs were aerodynamically inert at release.

However, aircraft speed has increased considerably. The aerodynamic loads increased with velocity squared and store separation became a problem. Weapon accuracy was reduced and aircraft damage was sustained.

Aircraft can now carry high density, externally stored weapons supersonically. It would also be advantageous to launch these weapons supersonically. The aerodynamic loads will be much larger and if large release disturbances are encountered, the damage to the aircraft can be severe.

One method of alleviating the store separation is to gyroscopically increase the weapon's inertial forces by inserting a spinning fly wheel. Admittedly, this is an added complication but it may be required to solve the problem. It is the purpose of this paper to present the mechanics of the internally stabilized weapon.

### Rigid Body Dynamics

From the linear theory of missile dynamics it can be shown that an approximate solution for the angle of attack of a spinning, symmetric, rigid body (neglecting damping) is

$$\alpha = K_1 e^{i\omega_1 t} + K_2 e^{i\omega_2 t} \quad (1)$$

If  $\alpha_0 = 0$  at  $t = 0$  then it may be shown that for an initial angular rate the maximum angle of attack is

$$|\alpha_{\max}| = 2|\dot{\alpha}_0| / [(p^2 I_x^2 / I_y^2) - 4M_{\alpha} / I_y]^{1/2} \quad (2)$$

Consequently, for statically stable configurations, the maximum angle of attack due to an angular rate may be reduced by increasing the angular momentum or by increasing the restoring moment. It is suggested that external stores be designed in accordance with Eq. (2).

The restoring moment for an external store is usually small due to the design conforming to space limitations. However its angular momentum may be increased considerably.

Two methods of increasing the angular momentum of external stores are readily apparent. The first would require spinning the store on the rack. This method would necessitate a complicated rack design which would probably not work for all stores. Moreover, the stores would be released with a high spin rate and be subject to Magnus instability.

A second method for increasing the angular momentum of an external store is to spin an internally mounted fly wheel. This method would work equally well for all types

of racks. It is interesting to note that a store with an internally spinning fly wheel, if dropped in the free fall mode, would not spin appreciably because the store and fly wheel fall at the same rate. Consequently, the torque due to friction coupling could not act efficiently. It is conceivable that the spin of the store could be optimized for stability by using fin cant or roll tabs.

Frick<sup>1</sup> of the Naval Weapons Laboratory has derived the equations of motion for two rigid bodies coupled by a bearing. In a body fixed reference frame (rolling with the outer body) the six-degree-of-freedom equations are:

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = (1/m) \begin{bmatrix} F(x_B) \\ F(y_B) \\ F(z_B) \end{bmatrix} + \begin{bmatrix} rv - qw \\ p_1 w - ru \\ qu - p_1 v \end{bmatrix} \quad (3)$$

$$M_{x_B} = I_{x_1} \dot{p}_1 + I_{x_2} \dot{p}_2$$

$$M_{y_B} = \tilde{I} \dot{q} - (\tilde{I} - I_{x_1} p_1 r + I_{x_2} p_2 r) \quad (4)$$

$$M_{z_B} = \tilde{I} \dot{r} + (\tilde{I} - I_{x_1} p_1 q - I_{x_2} p_2 q)$$

where

$$\tilde{I} = I_{y_1} + I_{y_2} + m_1 x_1^2 + m_2 x_2^2 \quad (5)$$

For aeroballistic axes: (do not spin with outer body):

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = (1/m) \begin{bmatrix} F_{x_A} \\ F_{y_A} \\ F_{z_A} \end{bmatrix} + \begin{bmatrix} rv - qw \\ -ru \\ qu \end{bmatrix} \quad (6)$$

$$M_{x_A} = I(x_1 \dot{p}_1 + I_{x_2} \dot{p}_2)$$

$$M_{y_A} = \tilde{I} \dot{q} + (I_{x_1} p_1 + I_{x_2} p_2) r \quad (7)$$

$$M_{z_A} = \tilde{I} \dot{r} - (I_{x_1} p_1 + I_{x_2} p_2) q$$

It can be shown that, for linear aerodynamics, an approximate solution for the angle of attack is

$$\alpha = \sum_{j=1}^2 K_j e^{\phi_j t} \quad (8)$$

where

$$\phi_j = \lambda_j + i\omega_j \quad (9)$$

$$\omega_{1,2} = [p_1 I_{x_1} + p_2 I_{x_2}] [1 \pm 1/\tau] / 2\tilde{I} \quad (10)$$

$$\tau = 1/(1 - 1/s)^{1/2} \quad (11)$$

$$s = [p_1 I_{x_1} + p_2 I_{x_2}]^2 / 4\tilde{I} M_\alpha \quad (12)$$

$$\lambda_{1,2} = Z_\alpha (1 \mp \tau) / 2m v + (M_q + M_{\dot{\alpha}}) (1 \pm \tau) / 2\tilde{I} \quad (13)$$

$$\pm M_{p_\alpha} p_1 \tau / [p_1 I_{x_1} + p_2 I_{x_2}]$$

The necessary and sufficient conditions for stability are:

$$\begin{aligned} s &> 1, \\ \text{or} \quad \lambda_{1,2} &< 0 \\ s &< 0 \end{aligned} \quad (14)$$

The maximum angle of attack due to an angular rate now becomes:

$$|\alpha_{MAX}| = 2 |\dot{\alpha}_0| / [p_1 I_{x_1} + p_2 I_{x_2}]^2 / \tilde{I}^2 - 4M_\alpha / \tilde{I}]^{1/2} \quad (15)$$

Consequently, the maximum angle of attack due to an angular rate is reduced by increasing the angular momentum of either the inner or outer bodies and the stability of the store depends on Eqs. (13-15).

### Discussion

The fly wheel complicates the design of an external store. However, folding fins, parachutes, etc. also complicate the design and are not always effective in improving its over-all performance. The over-all performance of a store would be improved according to Eqs. (12-15) if a properly designed fly wheel were inserted.

The fly wheel concept might be employed successfully to reduce the launch disturbance of low density stores, which have always been troublesome, or of high density stores at supersonic speeds. It could also be useful to improve the stability of marginally stable stores or to gyroscopically stabilize stores which are statically unstable.

### References

- <sup>1</sup>Frick, C. H., "Equations of Motion for Two Rigid Bodies Coupled by a Bearing," Rept. 1630, Nov. 25, 1958, Naval Proving Ground, Dahlgren, Va.

## Errata

### Reduction of ILS Errors Caused by Building Reflections

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[J. Aircraft 10, 167-171 (1973)]

ON p. 171, the figure numbers referred to as "4A," "4B," and "4C" in the paragraph immediately preceding the Conclusion section should read as "5A," "5B," and "5C."

Received April 26, 1973.

Index category: Air Navigation, Communication, and Traffic Control Systems.